

# Determining the gluon PDF

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Physics 290e

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# Outline

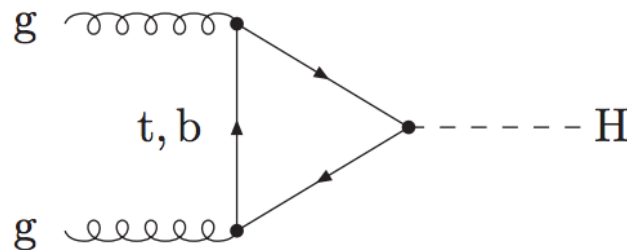
- What are PDFs? Why are they important?
- What do we learn about  $g(x, \mu^2)$  from deep inelastic scattering experiments?
- What do we learn about  $g(x, \mu^2)$  from hadron-hadron colliders?
- How do we get PDFs from data?

# Parton distribution functions

- Parton distribution functions (PDFs) describe the content of the proton
- PDFs depend on:
  - Bjorken  $x$ : the fraction of the proton momentum carried by the parton
  - Scale  $\mu^2$  ( $Q^2$ ): what you see inside the proton depends on the energy you probe it with
- The gluon PDF  $g(x, \mu^2)$  gives the probability that the proton contains a gluon with momentum fraction  $x$  at scale  $\mu^2$

# Why do we need PDFs?

- To make predictions!
- Example: Higgs production by gluon fusion
  - This is the main production mechanism for a Higgs at the LHC



- To calculate the cross section for this process in pp collisions, we need to know the gluon PDF

# Deep Inelastic Scattering

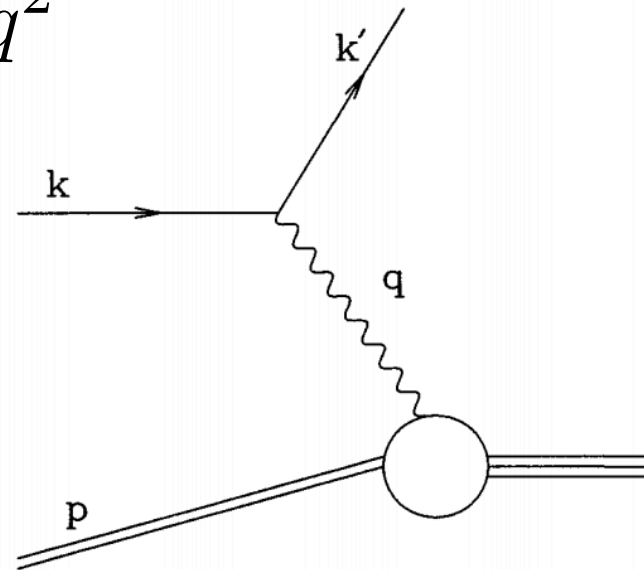
- Know  $p$  and  $k$  (from your beam/target)
- Measure  $k'$
- This is enough to determine all of the following, with  $Q^2 = -q^2$

$$M^2 = p^2$$

$$\nu = p \cdot q = M(E' - E)$$

$$x = \frac{Q^2}{2\nu} = \frac{Q^2}{2M(E - E')}$$

$$y = \frac{q \cdot p}{k \cdot p} = 1 - E'/E ,$$

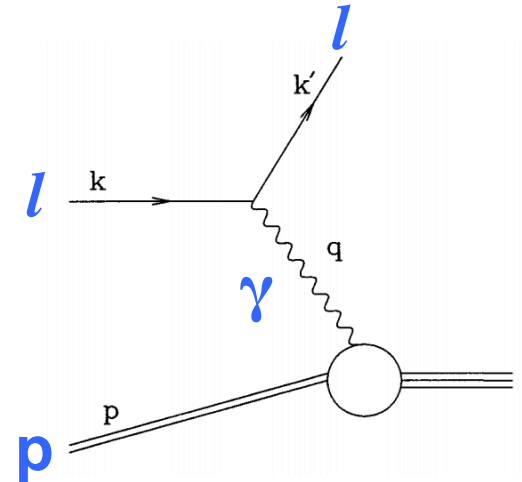


# Deep Inelastic Scattering

- Charged lepton scattering

$$lp \rightarrow lX$$

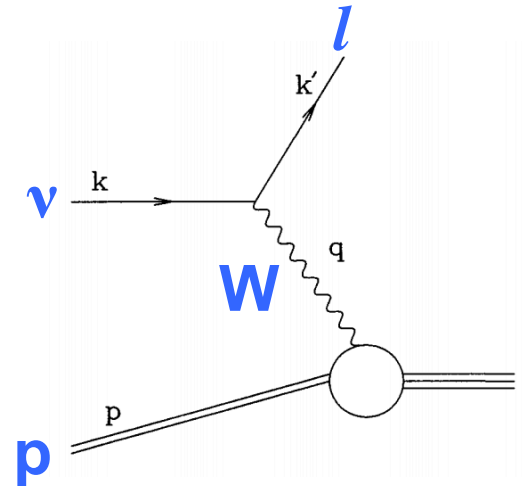
$$\frac{d^2\sigma^{em}}{dxdy} = \frac{8\pi\alpha^2 ME}{Q^4} \left[ \left( \frac{1 + (1-y)^2}{2} \right) 2xF_1^{em} + (1-y)(F_2^{em} - 2xF_1^{em}) - (M/2E)xyF_2^{em} \right]$$



- Neutrino scattering

$$\nu p \rightarrow lX$$

$$\frac{d^2\sigma^{\nu(\bar{\nu})}}{dxdy} = \frac{G_F^2 ME}{\pi} \left[ \left( 1 - y - \frac{M}{2E}xy \right) F_2^{\nu(\bar{\nu})} + y^2 x F_1^{\nu(\bar{\nu})} + (-) y \left( 1 - \frac{1}{2}y \right) x F_3^{\nu(\bar{\nu})} \right]$$



# Deep Inelastic Scattering

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**Structure Functions**  
 $F_i(x, Q^2)$

- Neutrino scattering

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# Determining PDFs from DIS

- Bjorken limit:  $Q^2 \rightarrow \infty$ 
  - In this limit, the parton momentum is parallel to the proton momentum
- The structure functions are sensitive to the quark PDFs

$$F_2^{em}(x) = 2xF_1^{em}(x) = \sum_{q,\bar{q}} e_q^2 x q(x)$$

- Can we learn anything about the gluon PDF from these experiments?

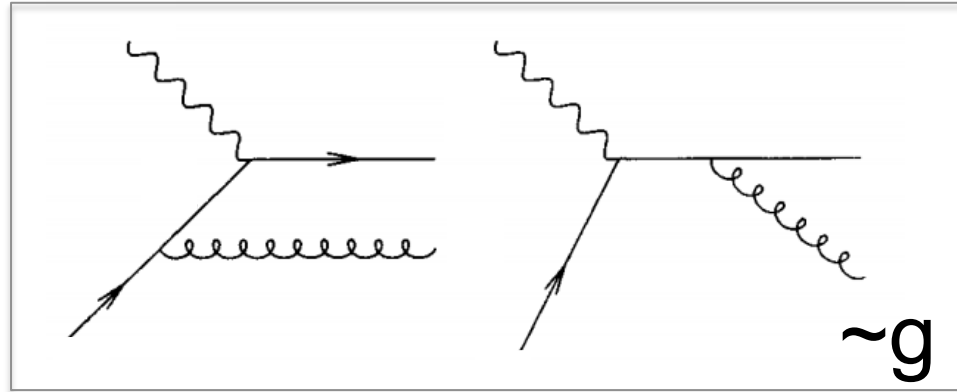
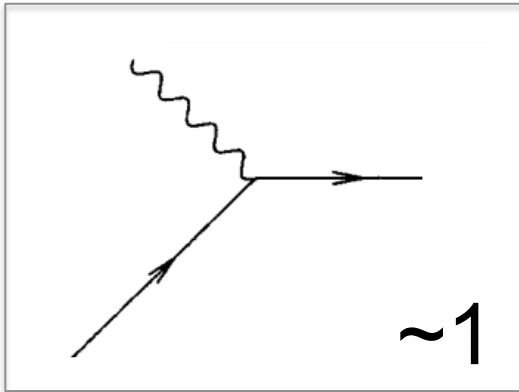


# Determining PDFs from DIS

- To learn about the gluon PDF, we must consider higher order in perturbation theory
  - Allow quarks to emit a gluon
- The Bjorken limit no longer applies
  - Gluon emission allows quarks to acquire momentum perpendicular to proton momentum
  - Scaling violation: must consider dependence of structure functions (and PDFs) on  $Q^2$
- Calculate the structure functions to first order in  $\alpha_s \sim g^2$  for ep collisions

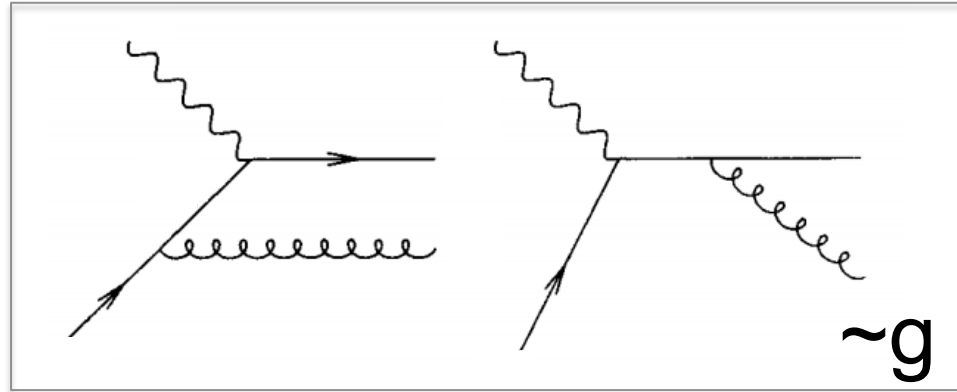
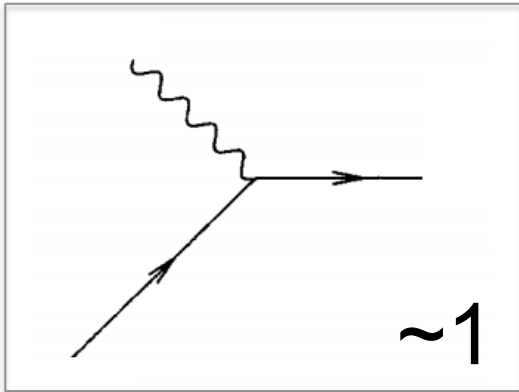
# QCD corrections to DIS

- Contributing diagrams for  $\gamma q \rightarrow qX$

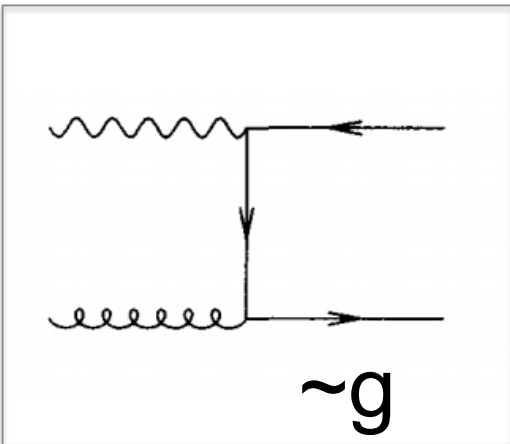


# QCD corrections to DIS

- Contributing diagrams for  $\gamma q \rightarrow qX$



- Contributing diagrams for  $\gamma g \rightarrow qX$



Through  $\gamma g \rightarrow q \bar{q}$  we are sensitive to the gluon PDF!

# QCD corrections to DIS

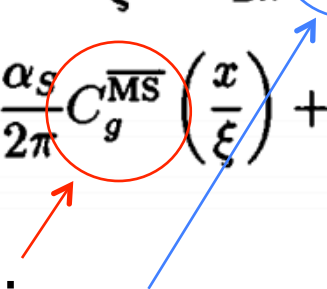
- Now just do QFT! Or consult a book...
  - R. K. Ellis, W. J. Stirling, and B. R. Webber, *QCD and collider physics*. Cambridge Univ. Press, Cambridge, UK, 1996.
- Find for electron-proton scattering:

$$F_2(x, Q^2) = x \sum_{q, \bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} q(\xi, Q^2) \left\{ \delta\left(1 - \frac{x}{\xi}\right) + \frac{\alpha_S}{2\pi} C_q^{\overline{\text{MS}}} \left(\frac{x}{\xi}\right) + \dots \right\} \\ + x \sum_{q, \bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} \underline{g(\xi, Q^2)} \left\{ \frac{\alpha_S}{2\pi} C_g^{\overline{\text{MS}}} \left(\frac{x}{\xi}\right) + \dots \right\} .$$

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$$+ x \sum_{q, \bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} \underline{g(\xi, Q^2)} \left\{ \frac{\alpha_s}{2\pi} C_g^{\overline{\text{MS}}} \left(\frac{x}{\xi}\right) + \dots \right\} .$$


Coefficient functions

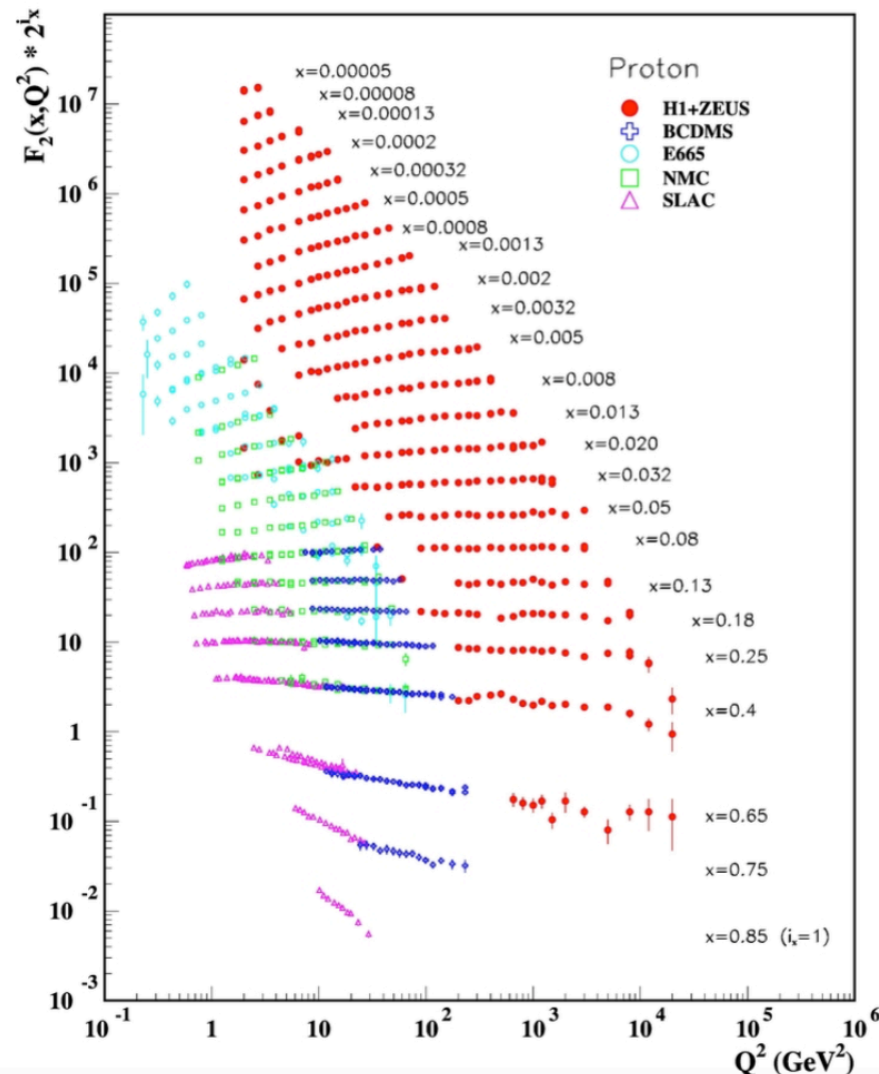
(known. depend on renormalization scheme, etc.)

# QCD corrections to DIS

$$F_2(x, Q^2) = x \sum_{q, \bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} q(\xi, Q^2) \left\{ \delta\left(1 - \frac{x}{\xi}\right) + \frac{\alpha_s}{2\pi} C_q^{\overline{\text{MS}}} \left(\frac{x}{\xi}\right) + \dots \right\} \\ + x \sum_{q, \bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} \underline{g(\xi, Q^2)} \left\{ \frac{\alpha_s}{2\pi} C_g^{\overline{\text{MS}}} \left(\frac{x}{\xi}\right) + \dots \right\} .$$

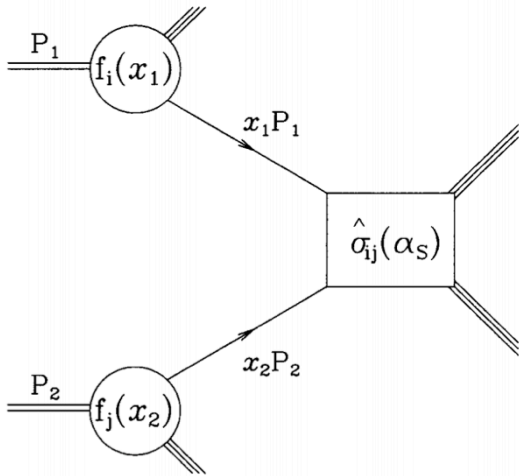
- We do learn something about the gluon PDF from DIS experiments!
- But this is not ideal:
  - Gluon PDF does not show up at leading order

# Summary of DIS Experiments



- Can see the dependence of the structure function  $F_2$  on  $x$  and  $Q^2$
- PDFs are extracted from cross section measurements
  - e.g. H1 and ZEUS at the ep collider HERA

# PDFs at hadron-hadron colliders



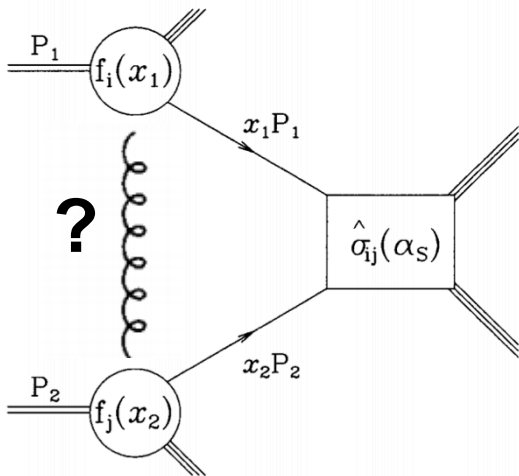
- Total cross section can be determined from PDFs and cross section of underlying process

$$\sigma(P_1, P_2) = \sum_{i,j} \int dx_1 dx_2 f_i(x_1, \mu^2) f_j(x_2, \mu^2) \hat{\sigma}_{ij}(p_1, p_2, \alpha_S(\mu^2), Q^2/\mu^2).$$



# PDFs at hadron-hadron colliders

- If we measure PDFs in ep and pp collisions, do we expect them to agree?
  - Do strong interactions between hadrons distort the PDFs?



# PDFs at hadron-hadron colliders

- If we measure PDFs in ep and pp collisions, do we expect them to agree?
  - Do strong interactions between hadrons distort the PDFs?
- These interactions give corrections  $\sim$  powers of  $m^2/E_{\text{CM}}^2$ 
  - Ok to neglect these at high energies
- So PDFs will be the same in ep and high energy pp experiments

# PDFs from jet production

- Processes resulting in two jets:

$$qq \rightarrow qq \qquad \bar{q}\bar{q} \rightarrow \bar{q}\bar{q}$$

$$q\bar{q} \rightarrow gg \qquad q\bar{q} \rightarrow q\bar{q}$$

$$gq \rightarrow gq \qquad g\bar{q} \rightarrow g\bar{q}$$

$$gg \rightarrow q\bar{q} \qquad gg \rightarrow gg$$

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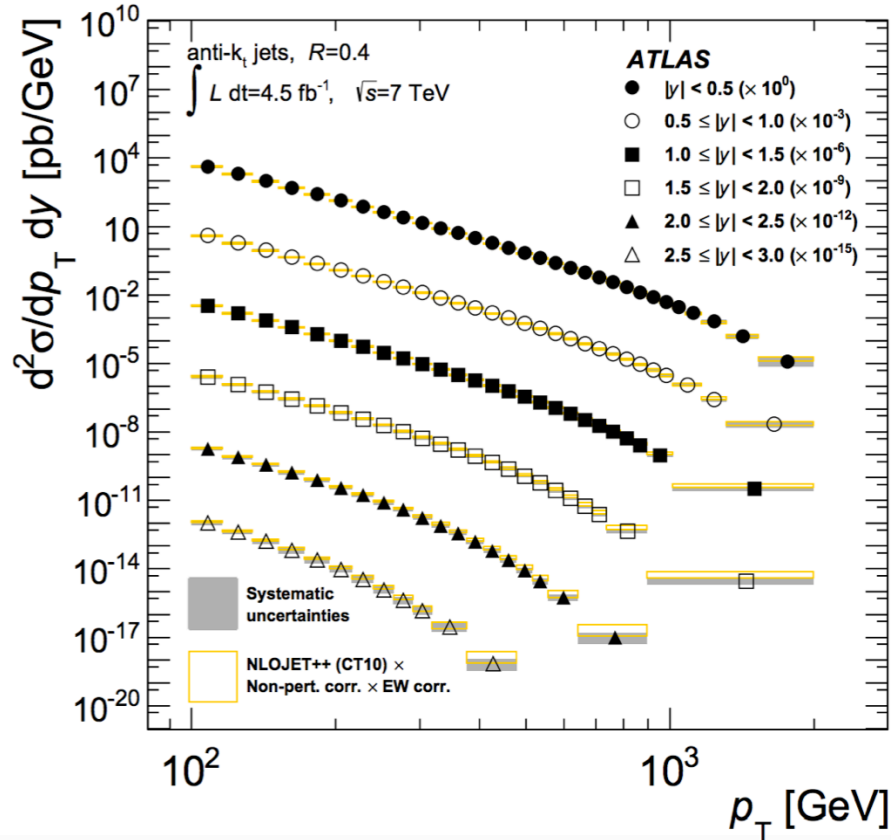
$$q\bar{q} \rightarrow gg \quad q\bar{q} \rightarrow q\bar{q}$$

$gq \rightarrow gq$	$g\bar{q} \rightarrow g\bar{q}$	Initial states with gluons
$gg \rightarrow q\bar{q}$	$gg \rightarrow gg$	

- The gluon PDF enters at leading order!
- Can experimentally measure inclusive jet cross section and learn about  $g(x, \mu^2)$

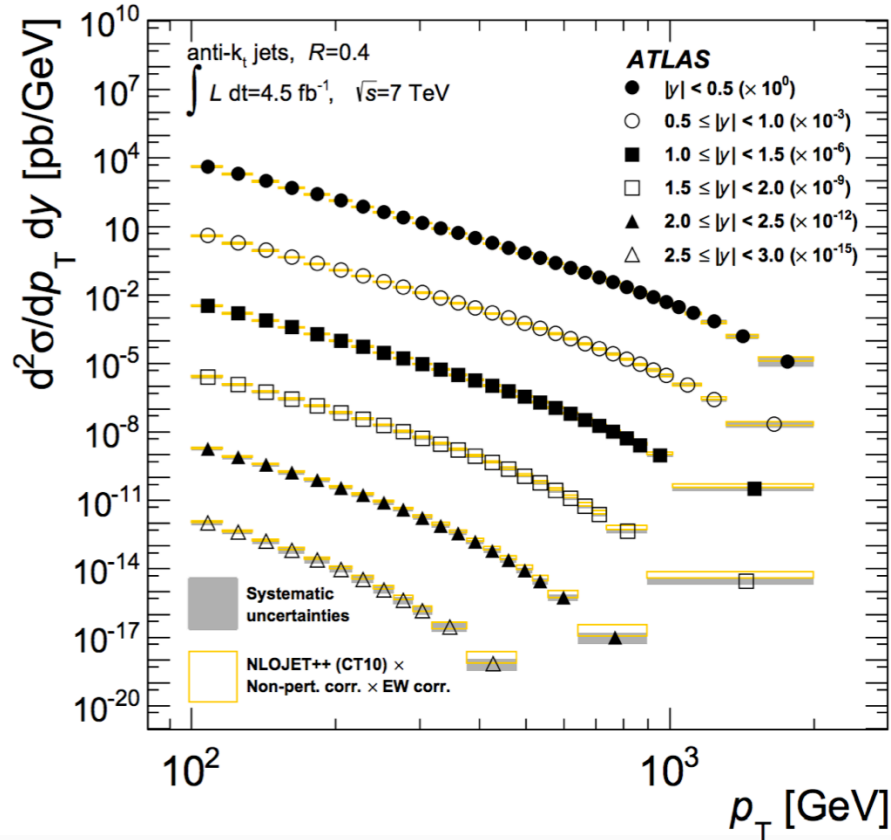
# Inclusive jet cross section

- Why inclusive?



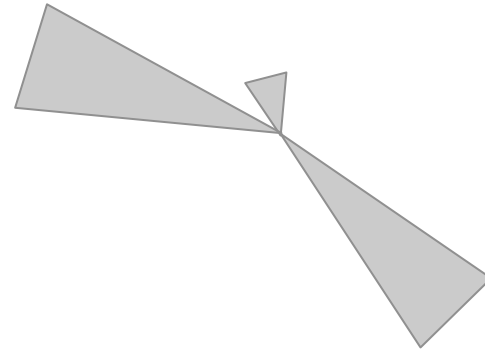
Measurement from  
ATLAS at 7 TeV

# Inclusive jet cross section



Measurement from  
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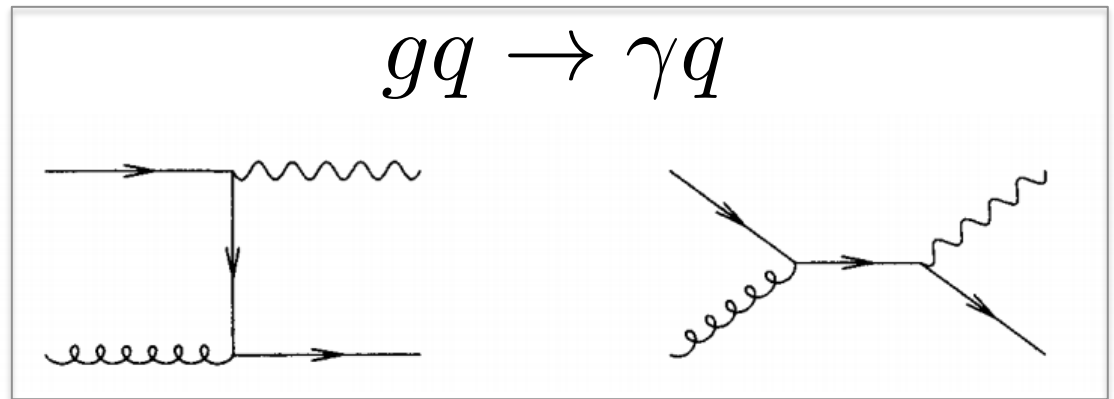
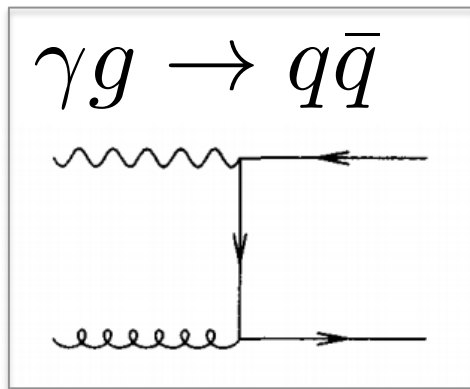
- Why inclusive?
- Is this 2 jets or 3?



- For the inclusive cross section, it doesn't matter if you're wrong

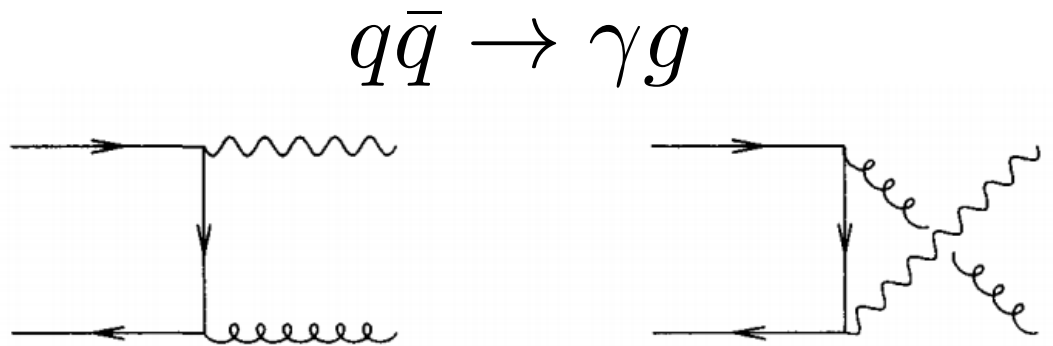
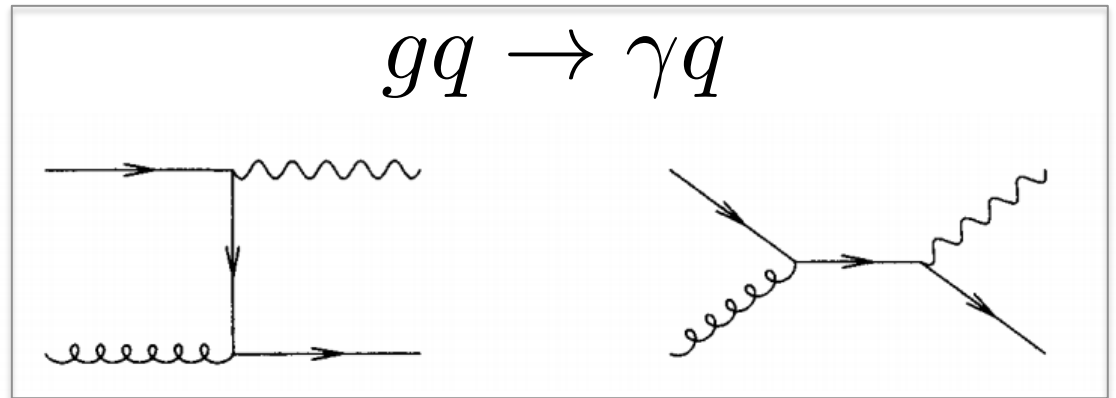
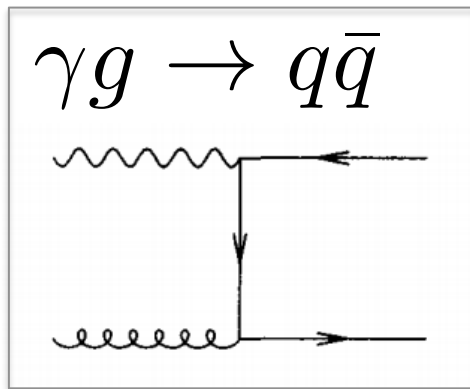
# Prompt photon production

- Try flipping the diagram that was most useful for DIS!



# Prompt photon production

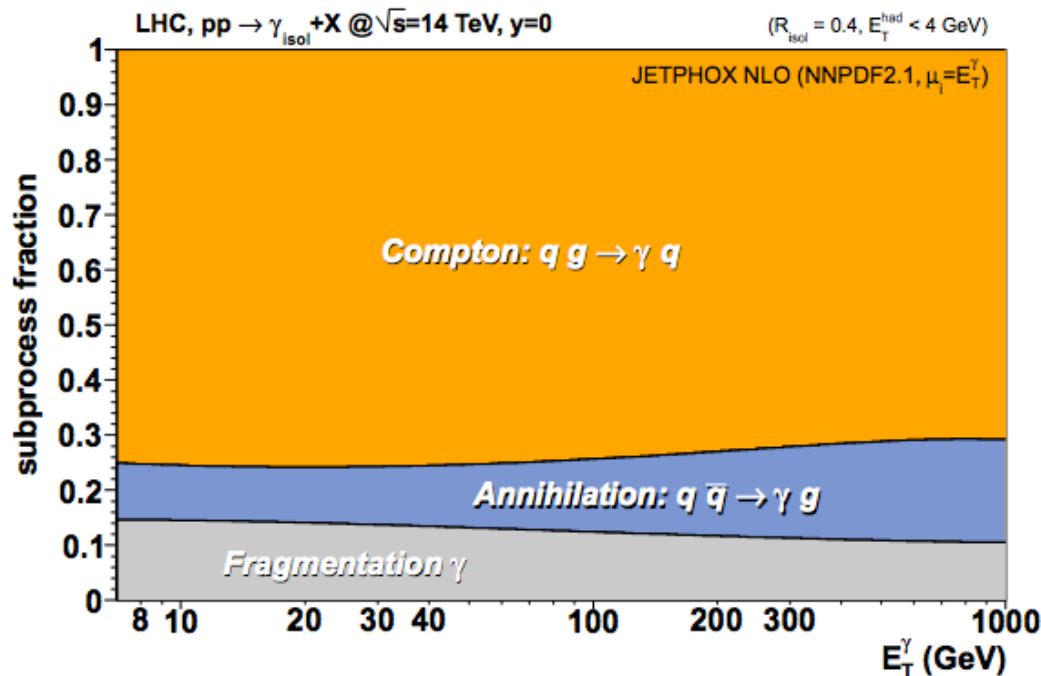
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# Prompt photon production

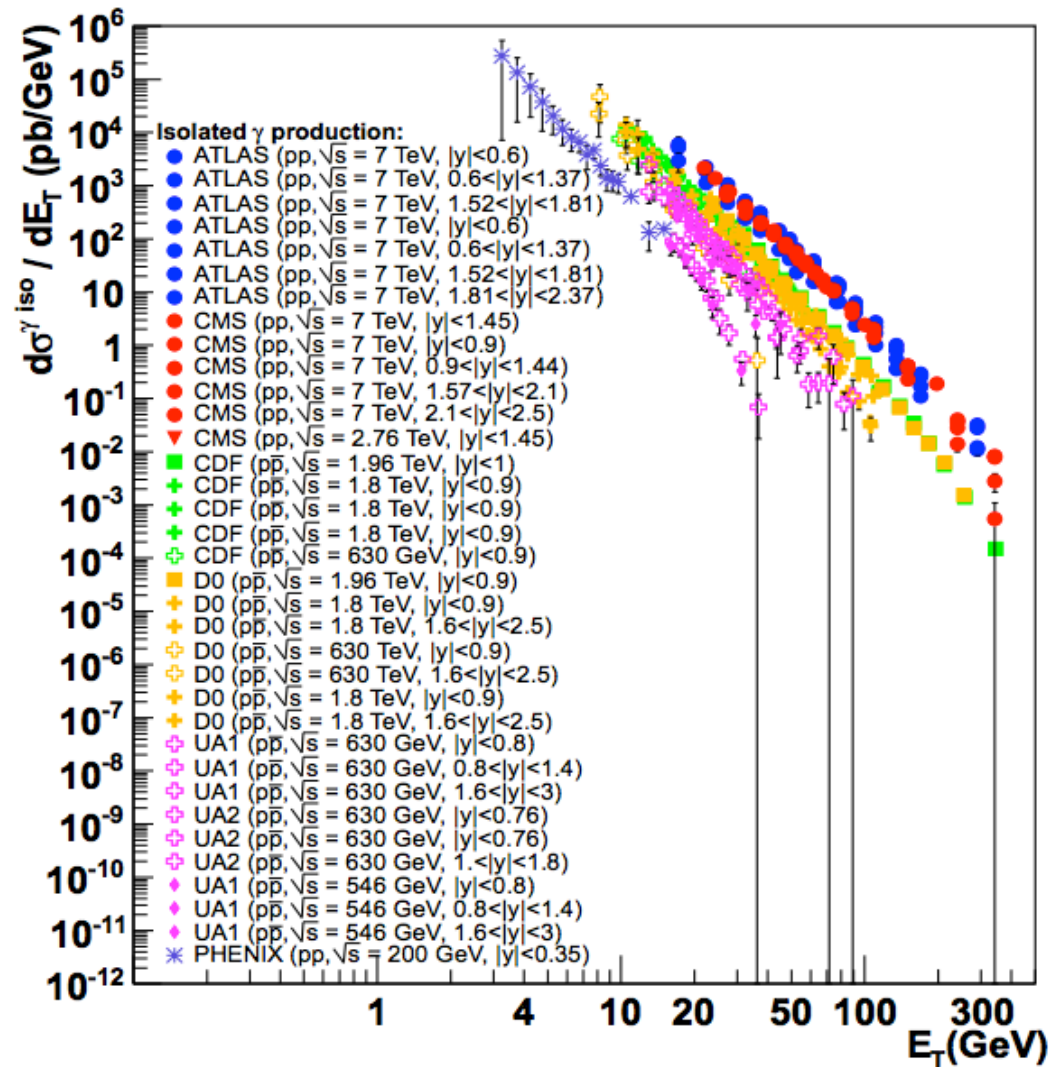
- Relative contribution of prompt photon processes at the LHC at 14 TeV
  - The useful one dominates!



# ATLAS prompt $\gamma$ search at 7 TeV

- Look for events containing one photon
- The photon must be
  - Energetic enough: photon  $E_T > 100$  GeV
  - Isolated: extra  $E_T$  inside a cone of size  $\Delta R = 0.4$  around the photon must be  $< 7$  GeV
  - In the barrel or end-cap ( $|\eta| < 1.37$  or  $1.52 < |\eta| < 2.37$ ), where the electromagnetic calorimeter can measure the energy most accurately
- Main background is light mesons decaying to photons, such as  $\pi^0$

# Measurements of prompt photon cross section at hadron-hadron colliders



# Extracting PDFs from data

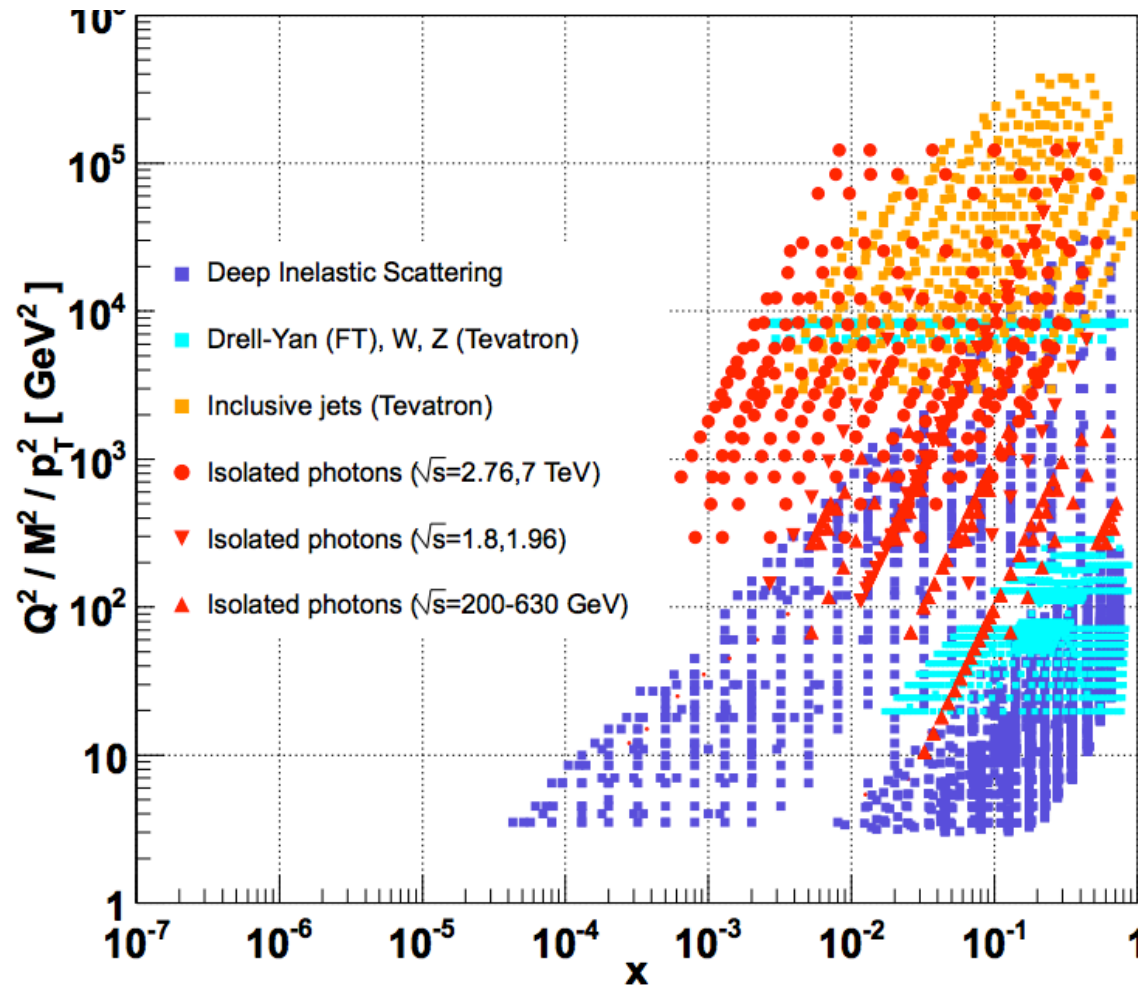
- Assume a functional form for the PDFs, and then fit to cross section data

$$f_i(x, \mu^2) = A_0 x^{A_1} (1 - x)^{A_2} P_i(x; A_3, \dots)$$

for  $P_i$  some smooth function

- I won't talk about the statistics magic that is required to accomplish:
  - Fitting to data from many experiments
  - Getting uncertainties out of these fits

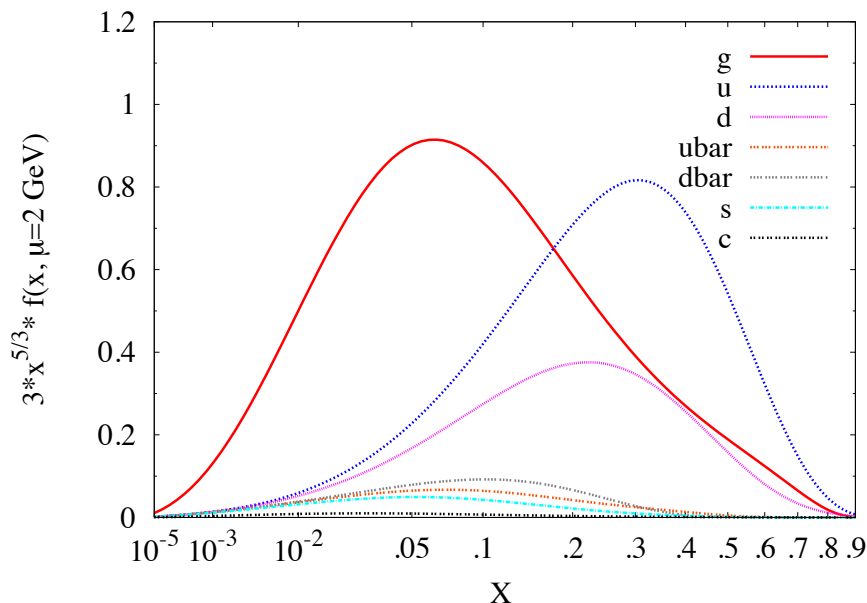
Regions in  $x$ ,  $Q^2$  ( $\mu^2$ ) where different experiments are useful for determining PDFs



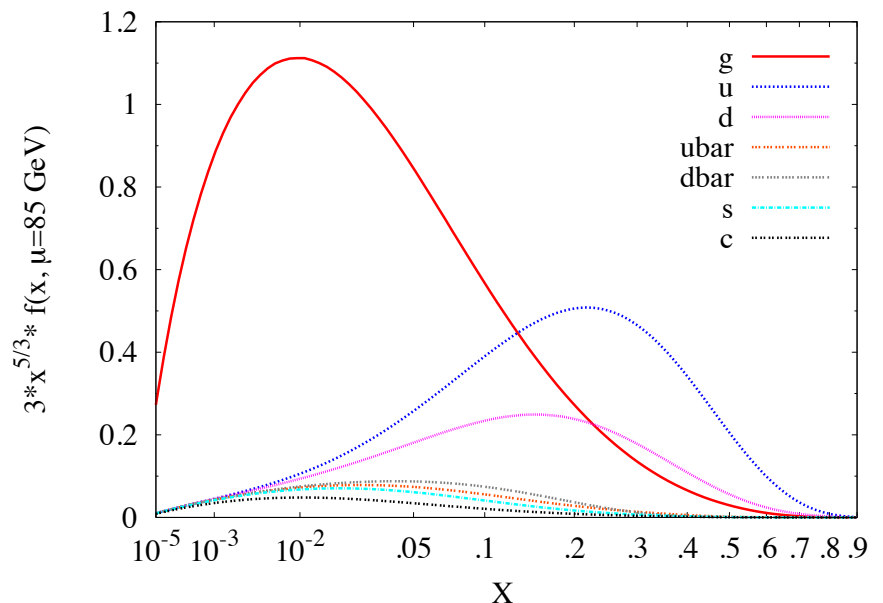
# Modern PDF sets

- Many groups have done these fits and put forward sets of PDFs
- All find that the gluon dominates at low  $x$ , especially at high  $\mu^2$

CT10.00 PDFs (area proportional to momentum fraction)



CT10.00 PDFs (area proportional to momentum fraction)



# Modern PDF sets

- Many PDF sets are available here for use in Monte Carlo simulations:
  - <https://lhapdf.hepforge.org/pdfsets.html>
- These PDF sets differ in many ways:
  - Functional form of the PDFs
  - Number of parameters in the fit
  - What data is used in the fits
  - Order in perturbation theory
  - Details of the statistical methods

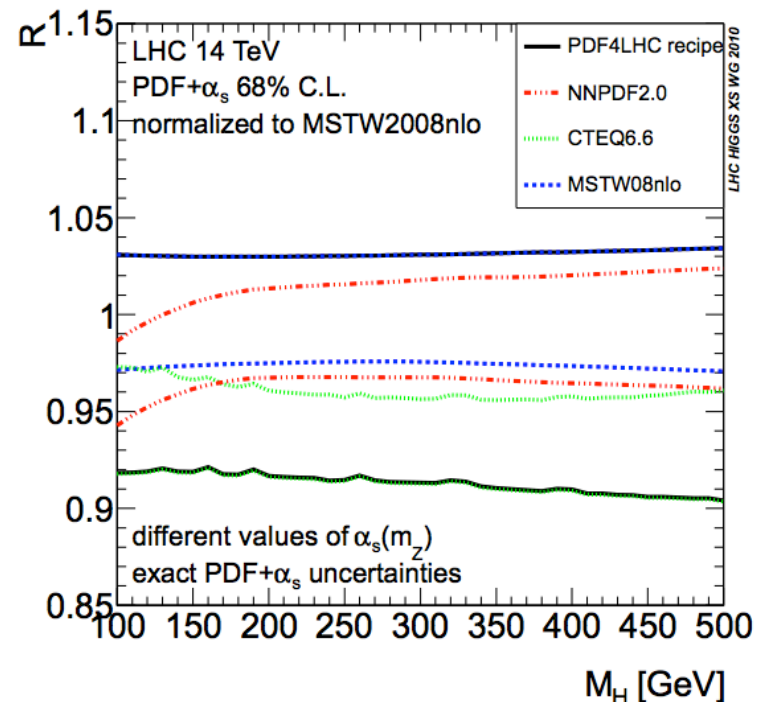
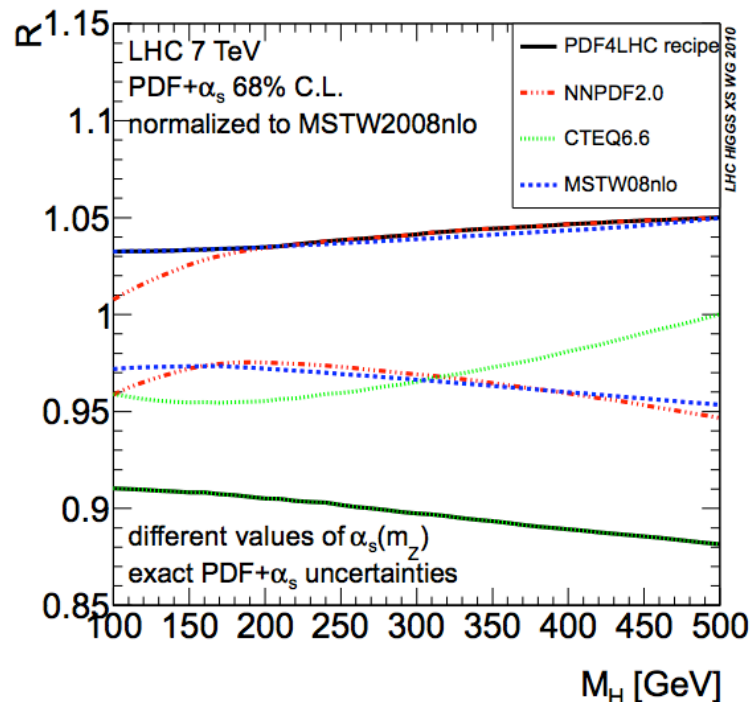
# PDF uncertainties

- Some factors contributing to the PDF uncertainty:
  - Uncertainties from input data from many experiments (not always consistent!)
  - Uncertainty on values of physical parameters (especially  $\alpha_S$ , also heavy quark masses, etc.)
  - Order in perturbation theory



# PDF uncertainties: an example

- Uncertainties on the cross section of Higgs production via gluon fusion
- Bands shown are combined PDF+ $\alpha_s$  uncertainty for several PDF sets



# Summary

- We need PDFs (and uncertainties) if we want to make accurate predictions!
- We get information about the gluon PDF by measuring cross sections of
  - Deep inelastic scattering
  - Jet production
  - Prompt photon production
- Modern PDF sets are obtained by fitting data from these experiments

# References

- R. K. Ellis, W. J. Stirling, and B. R. Webber, *QCD and collider physics*. Cambridge Univ. Press, Cambridge, UK, 1996.
- <http://xxx.lanl.gov/pdf/1301.6754v1>
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- [http://web.hep.uiuc.edu/atlas/OA\\_Software/QuarksAndGluonsReviewPaper.pdf](http://web.hep.uiuc.edu/atlas/OA_Software/QuarksAndGluonsReviewPaper.pdf)
- <http://xxx.lanl.gov/pdf/1202.1762v2>
- <https://arxiv.org/pdf/1410.8857v3>
- <https://arxiv.org/pdf/1311.1440v1>